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SELFSIMILAR SOLUTIONS OF THE PROBLEM OF HEAT AND MASS TRANSFER IN A SATURATED POROUS MEDIUM WITH A VOLUME HEAT SOURCE*

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Selfsimilar solutions of a system of stationary equations of heat condunction and filtration of molten material in the presence of a volume heat source generated by absorption of the energy of electromagnetic radiation, are considered. The possibility of the existence of a selfsimilar solution in the case of various (plane, cylindrical and spherical) spatial symmetries is studied. The existence of a selfsimilar solution is shown for the axisymmetric case when the radiation obeys a prescribed law. The influence of the surface volume heating and convective heat transfer due to filtration is studied. A solution for the case when the filtration of the molten phase is quasistationary is also investigated.

1. Let the pores of a solid (first component) be filled, at first, with a very viscous (becoming solid in the limit) medium (second component), which becomes, when heated, progressively less viscous (diluted) and tends to expand due to the action of heat conduction and absorption of high frequency electromagnetic radiation (HFER) (of frequency 10^{-1}—10^{-2} MHz). The dilute component may flow under the action of a pressure difference relative to the fixed porous solid, e.g. in a well. Such a process can be utilized to extract highly viscous (bituminous) oils /1-4/, gas hydrates, in drying and purifying porous materials, etc.

The thermodynamics and hydrodynamics of the process of heating a saturated medium is studied using the methods of the mechanics of continuous media, taking into account a possible phase transition of the first kind, of the melting or solidifying type, for the saturating (second) component, under the following basic assumptions.

1°. The process of melting takes place at a geometrical surface, i.e. at the melting front of zero thickness (Stefan's idea of the melting process).

2°. Outside the melting front (a surface of strong discontinuity) the distances at which the parameters of the medium very substantially (e.g. the characteristic lengths of the HFER energy absorption zone) are much greater than the characteristic pore sizes, and the distances between them, which in turn are much greater than the molecular and kinetic dimensions (e.g. the molecular mean free path).

3°. The temperature of the phase and components in each elementary volume of the porous medium are the same.

4°. The motion of the liquid (molten) phase in the porous medium is inertialess and obeys D'Arcy's law.

5°. There is no volume change in the solid phase and no deformation of the skeleton of the porous medium.

The first assumption can be used when the size of the phase transition (melting) zone is very much less than the wavelength under investigation and the characteristic length of the problem (e.g. the characteristic length of the HFER energy absorption zone). Under the second assumption we can study the dynamics of the system consisting of a porous solid (the first component) filled with a liquid (molten) and solid (non-molten) phase of the second component outside the melting front, within the framework of a model of three interpenetrating and interacting continuous media: 1) the liquid (molten) phase of the second component, 2) the solid (non-molten) phase of the second component, 3) the first component, i.e. the skeleton of the rock. Furthermore, the parameters corresponding to these media are denoted by the indices $i = 1, 2, 3$; the indices m, b denote the parameters of the medium at the melting front and at the well boundary; α_i is the volume fraction of the i-th phase, T is the temperature, m is the porosity, x is the spatial coordinate, x_b is the well radius (the size of the HFER source), $x_m(t)$ is the coordinate of the melting front in motion, and t is the time.

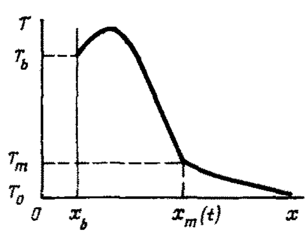

Fig.1

Thus in accordance with the above assumptions and notation,

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the front $x_m(t)$ at which melting occurs will represent a boundary separating the zone of the solid porous skeleton (third phase) filled with the molten second component (first phase): $x_b < x < x_m(t), \alpha_1 = m, \alpha_2 = 0, \alpha_3 = 1 - m, T > T_m$ and the zone of porous solid filled with the solid second component (second phase): $x > x_m(t), \alpha_1 = 0, \alpha_2 = m, \alpha_3 = 1 - m$, i.e. in the present scheme we have no zone containing a mixture of the molten and solid second component (the first and second phase): $\alpha_1\alpha_2 = 0$. The characteristic temperature distribution occurring when the medium is heated, due to heat conduction and absorption of the HFER energy is shown in Fig.1 (the radiation source is situated at the origin of coordinates).

The third assumption enables us to describe the process in question within the framework of a single temperature model. This is due to the fact that in the majority of important practical cases the characteristic time of smoothing the phase temperatures $(10^{-1}-10$ sec) is much shorter than the characteristic time needed to heat the medium by means of an external heat source and the characteristic time of the hydrodynamic process $(10^4-10^5$ sec). The latter assumption can be adopted when the displacement of the melting front caused by the expansion of the solid before the front is small compared with the displacement of this surface caused by the phase transition. In this case the problem simplifies, since the temperature field before the melting front can be determined without solving the equation of motion.

Under the above assumptions, the equations of continuity, of filtration of the phases and of heat influx (heat conduction) of the mixture outside the surface of strong discontinuity (the melting front $x_m(t)$) can be written, for the case of one-dimensional symmetric (plane $\nu = 0$, cylindrical $\nu = 1$ and spherical $\nu = 2$) motion in the Eulerian (x, t)-coordinate system in the following form /5-7/:

$$\frac{\partial \rho_i}{\partial t} + \frac{1}{x^\nu}\frac{\partial}{\partial x}(x^\nu \rho_i v_i) = 0, \quad i = 1, 2, 3 \tag{1.1}$$

$$\alpha_1 v_1 = -\frac{k}{\mu_1}\frac{\partial p_1}{\partial x}, \quad v_2 = v_3 = 0 \tag{1.2}$$

$$\rho c \frac{\partial T}{\partial t} + \alpha_1 \rho_1 c_1 v_1 \frac{\partial T}{\partial x} = \frac{1}{x^\nu}\frac{\partial}{\partial x}\left(\lambda x^\nu \frac{\partial T}{\partial x}\right) + A + Q \tag{1.3}$$

$$\alpha_1 + \alpha_2 + \alpha_3 = 1, \quad \alpha_1 + \alpha_2 = m \tag{1.4}$$

$$\alpha_1\alpha_2 = 0 \tag{1.5}$$

where $\rho_i, p_i, v_i, c_i, \mu_i, \lambda_i$ are, respectively, the density, pressure, velocity, viscosity, heat capacity and thermal conductivity of the i-th phase, k is permeability, A is the work done by internal forces and Q is the intensity of the volume heat source. Moreover, the heat capacity c and thermal conductivity λ of the mixture are additive with respect to the masses and volumes of the phases respectively:

$$\rho c = \sum_{i=1}^{3} \alpha_i \rho_i c_i, \quad \lambda = \sum_{i=1}^{3} \alpha_i \lambda_i \tag{1.6}$$

Henceforth, we shall consider the case in which the contribution of the work done by internal forces A towards the temperature change can be neglected $(A \ll Q)$.

In order to close the system of differential Eqs.(1.1)-(1.3) we use the following equations of state of the phases:

$$\rho_1 = \rho_{10}[1 + \beta_p(p_1 - p_0)], \quad \rho_2 = \text{const}, \quad \rho_3 = \text{const} \tag{1.7}$$

where β_p is the compressibility of the liquid phase.

When the pressure drops $\Delta p \lesssim 10^2$ MPa and $\beta_p \sim 10^{-3}$ MPa^{-1}, we have

$$\frac{v_1(\partial\rho_1/\partial x)}{\rho_1(\partial v_1/\partial x)} \sim \beta_p \Delta p \ll 1$$

From this it follows that the convection component of the liquid phase density change can be neglected. Then from (1.1) we obtain, taking into account (1.7), the following linear equation of piezoconduction:

$$x_b < x < x_m(t)$$
$$\frac{\partial p_1}{\partial t} - \frac{\varkappa}{x^\nu}\frac{\partial}{\partial x}\left(x^\nu \frac{\partial p_1}{\partial x}\right) = 0, \quad \varkappa = \frac{k}{m\mu_1\beta_p} \tag{1.8}$$

where $\varkappa$ is the coefficient of piezoconduction. The problem of the pressure distribution in front of the melting front $(x > x_m(t))$ refers, in the present formulation $(\rho_2 = \text{const}, \rho_3 = \text{const}, m = \text{const})$, to the class of statically undetermined problems, and the above distribution does not affect the distribution of the remaining parameters.

The distribution of the heat sources Q appearing as the result of the absorption of the HFER energy is found from Poynting's equation and the Bouguer-Lambert law for a monochromatic wave:

$$Q = -\nabla \cdot \mathbf{R}; \quad \nabla \cdot \mathbf{R} = R/L \quad (L > 0) \tag{1.9}$$

where $\mathbf{R}$ is the radiation intensity vector and L is the wavelength at which the medium in question absorbs the energy of the electromagnetic wave (EW). When a one-dimensional monochromatic wave (plane, cylindrical and spherical) propagates through a homogeneous isotropic medium, the equation for the radiation intensity and boundary condition at the well has the form

$$\frac{\partial}{\partial x}(x^\nu R) = \frac{x^\nu R}{L}; \quad x = x_b, \quad R = R_b = \frac{N^{(e)}}{S_b} \tag{1.10}$$

$$S_b = \chi(\nu)\, x_b^\nu; \quad \chi(0) = 1, \quad \chi(1) = 2\pi, \quad \chi(2) = 4\pi$$

where R_b is the radiation intensity at the boundary of the well $(x = x_b)$ given in terms of the intensity $N^{(e)}$ and surface area S_b of the radiator.

Generally, the absorption wavelength for the given medium is determined by the frequency ω of EW and depends on the pressure p and temperature (F.L. Sayakhov et al (see /1, 3/) carried out a large number of determinations of the electrophysical parameters necessary to calculate L for the materials used in petroleum technology; earlier, test measurements of these parameters were carried out in /4/). It was found that even at a fixed frequency ω the equation for the thermodynamic parameters (1.1)-(1.7) and equations for Q and the electrophysical parameter R were interrelated, i.e. they had to be solved together.

Often the effect of the pressure p and temperature on the absorption wavelength L can be neglected. then, for a fixed ω the quantity L will become an a priori known parameter which will give at once the radiation intensity R and volume heat source intensity Q independently of the solutions of the thermohydrodynamic equations:

$$Q = \frac{R_b}{L}\left(\frac{x_b}{x}\right)^\nu \exp\left(-\frac{x - x_b}{L}\right) \tag{1.11}$$

Remembering that the EW can be reflected from the melting surface $x_m(t)$, i.e. from the surface at which the electromagnetic properties of the medium become discontinuous, we can represent the volume heat source for the whole mixture, in the following form:

$$\begin{gathered} x_b < x < x_m(t) \\ Q_l = \frac{R_b}{L_l}\left(\frac{x_b}{x}\right)^\nu \left[\exp\left(-\frac{x - x_b}{L_l}\right) + H\exp\left(-\frac{x_m - x_b}{L_l} - \frac{x_m - x}{L_s}\right)\right] \\ x > x_m(t) \\ Q_s = (1 - H)\frac{R_l}{L_s}\left(\frac{x_b}{x}\right)^\nu \exp\left(-\frac{x_m - x_b}{L_l} - \frac{x - x_m}{L_s}\right) \\ \left(\frac{1}{L_l} = \frac{\alpha_1}{L_1} + \frac{\alpha_3}{L_3}, \frac{1}{L_s} = \frac{\alpha_2}{L_2} + \frac{\alpha_3}{L_3}\right) \end{gathered} \tag{1.12}$$

Here the parameter H $(0 \leqslant H \leqslant 1)$ characterizes the reflection of the wave in question from the interphase boundary, and is determined as the ratio of the energies of the reflected and incident wave. (We have $(H = 0$ for zero reflection, and $H = 1$ for the total reflection). We denote the parameters of the mixture (the absorption wavelength) in the regions of the molten $(x_b < x < x_m(t))$ and non-molten $(x > x_m(t))$ second component by the subscripts l and s respectively. We must also remember that $Q_k \geqslant 0$ $(k = l, s)$ always holds.

The system of Eqs.(1.2)-(1.8) (taking (1.12) into account) is closed. It can be used to study the laws governing the heating of a medium by heat conduction (a surface heat source q_b) and absorption of HFER energy (a volume heat source Q). The corresponding mathematical problem consists of obtaining a solution of the system of Eqs.(1.2)-(1.8) with the following initial and boundary conditions:

$$t = 0, \; T = T_0 \leqslant T_m \tag{1.13}$$

$$x = x_b, \; T = T_b \;\text{ or }\; \lambda_b S_b (\partial T/\partial x) = -q_b \tag{1.14}$$

$$m S_b \rho_{1b} v_{1b} = g_b \tag{1.15}$$

$$x \to +\infty, \; T \to T_0 \tag{1.16}$$

and conditions at the melting front $x_m(t)$:

$$v_{1m} = \left(1 - \frac{\rho_2}{\rho_{1m}}\right)\frac{dx_m}{dt}, \; \frac{dx_m}{dt} = \frac{j}{m\rho_2} \tag{1.17}$$

$$T = T_m = \text{const}, \ jl = q_l + q_s \tag{1.18}$$
$$q_l = -\lambda_l \frac{\partial T}{\partial x}\Big|_{x=x_m-0}; \quad q_s = \lambda_s \frac{\partial T}{\partial x}\Big|_{x=x_m+0}$$

Here g_b is the total mass flux of the liquid (first) phase, j, l is the intensity and specific heat of the phase transition, q_l, q_s are the heat fluxes arriving at the interphase surface from the direction of the moving and immobile phases, $q_b = q(x_b, t)$ is the intensity of the total heat flux across the boundary $x = x_b$ $(q_b) > 0$ corresponds to the case when heat is fed in, and $q_b < 0$ corresponds to the case when the heat is removed, $q_b = 0$ means that there is no heat conduction at the well boundary, and the subscript 0 characterizes the parameters of the initial state. Equations (1.17), (1.18) are obtained from the condition of mass balance and from the quasistatic (filtration) approximation to the energy balance at the interphase boundary $x_m(t)$. The specific heat of phase transition l representing the difference between the enthalpies of the phases, is used up by the change in internal energy of the medium and by the work done by the pressure forces during the phase transition.

We note that in /8/ the energy equation at the interphase boundary has redundant terms of the form $\rho_{1m}c_1v_{1m}T_m$, $m\rho_{1m}c_1T_m\,(dx_m/dt)$. A detailed discussion of the equation of conservation at the interphase boundaries is given in Chapter 2 of the book /7/.

In general, the liquid parameters of the first phase at the melting front must be determined from the condition of phase equilibrium along the line of melting, using the Clapeyron-Clausius equation for $T_m(p)$. Henceforth, we shall assume (see /9/) $T_m = \text{const}$.

2. Usually, the radius of the well is much less than the characteristic linear dimensions of the problem, and is therefore not realized. In this case the boundary conditions determining the heat fluxes q_b, liquid phase flow rate g_b and the influx of radiation R_b, are given in the form of the limiting relations $(x_b \to 0)$:

$$\lim\left[\lambda_b S_b(x_b)\frac{\partial T}{\partial x}\Big|_{x=x_b}\right] = -q_b \tag{2.1}$$
$$\lim\,[mS_b\rho_{1b}v_{1b}] = g_b \tag{2.2}$$
$$\lim\,[R_bS_b(x_b)] = N^{(e)} \quad (S_b = \chi(\nu)\,x_b^{\nu}) \tag{2.3}$$

We shall consider, for the system of equations given above, possible selfsimilar solutions depending on a single variable $z = x^2t^{\eta}$. The equation of heat conduction in the melting zone now takes the form

$$0 < z < z_m = (x_m(t))^2\,t^{\eta} \tag{2.4}$$
$$\lambda_l\frac{d^2T}{dz^2} + \left[\frac{\lambda_l(\nu+1)}{2z} - \frac{\rho c\eta}{4t^{\eta+1}} + V_l(z)\right]\frac{dT}{dz} + \frac{Q_l}{4zt^{\eta}} = 0$$
$$V_l(z) = \alpha_1\rho_1c_1\frac{k}{\mu_1}\frac{dp_1}{dz}, \quad \frac{Q_l}{4zt^{\eta}} = \frac{N^{(e)}}{4\chi(\nu)\,z^{1+\nu/2}}\times$$
$$\frac{t^{\nu}}{L_l}\left[1 + H\exp\left(-2\,\frac{\sqrt{z_m}-\sqrt{z}}{L_l t^{\eta/2}}\right)\right]\exp\left(-\frac{\sqrt{z}}{L_l t^{\eta/2}}\right)$$

For a selfsimilar solution to exist, it is necessary that all coefficients of this equation depend only on z and be independent of t. In particular, analysing the second term within the square brackets of the equation heat conduction, we find that $\eta = -1$. The analysis of the last term, connected with the absorption of the energy Q, yields $\nu = 1$, and

$$L_k = \sqrt{h_k t} \quad (h_k = \text{const}), \quad k = l, s \tag{2.5}$$

Therefore, in the discussion that follows, we shall confine ourselves to the axisymmetric case $(\nu = 1, \chi(\nu) = 2\pi)$. The law of variation of the absorption wavelength (2.5) can be obtained by varying the frequency ω of the wave in question with time.

We note that without the volume heat sources, i.e. in the case when the medium absorbs no EW energy $(Q = 0)$, we see from (2.4) that a selfsimilar solution of the problem in question, taking the filtering motion of the liquid phase of the second component with velocity $V_l(z)$ into account, can exist for any $\nu = 0, 1$, and here, as before, we have $\eta = -1$.

3. In order to analyse the equations given, it is best to introduce the following dimensionless variable parameters which, together with the coefficient of porosity m, determine the solutions of the problem in question:

$$\zeta = \frac{x^2}{a_l t}, \quad \theta = \frac{T}{T_m}, \quad P = \frac{p_1}{p_0}, \quad \delta_i = \frac{\rho_i}{\rho_{10}} \quad (i = 1, 2, 3) \tag{3.1}$$

$$\Phi_1 = \frac{\rho_{10}c_1}{(\rho c)_l}, \quad E_p = \beta_p p_0, \quad a_k^* = \frac{a_k}{a_l}, \quad \varkappa^* = \frac{\varkappa}{a_l} \tag{3.2}$$

$$h_k^* = \sqrt{\frac{h_i}{a_l}}, \quad K_k = \frac{N^{(e)}}{\chi(\nu)\lambda_i T_m}, \quad G_i = \frac{\Lambda_i}{a_l},$$

$$G_b = \frac{g_b}{m\chi(\nu)\rho_{1b}\varkappa B_p}$$

$$\left(a_k = \frac{\lambda_k}{(\rho c)_k}, \quad (\rho c)_k = \sum_{j=1}^{3} \alpha_j \rho_j c_j, \quad \Lambda_k = \frac{\lambda_k T_m}{\rho_2 l}, \quad k = l, s\right)$$

In accordance with (3.1), (3.2), Eq.(1.8) and boundary conditions (2.2) and (1.17) have the form

$$0 < \zeta < \zeta_m, \quad \frac{d}{d\zeta}\left(\zeta \frac{dP}{d\zeta}\right) + \frac{1}{4\varkappa^*}\frac{dP}{d\zeta} = 0, \quad \zeta = \zeta_m, \quad P = P_m \tag{3.3}$$

$$\zeta = 0, \quad \zeta\,(dP/d\zeta) = -G_b$$

(the constants ζ_m and P_m must be specified). The solution of this problem is determined by the following expression in terms of the exponential integral function:

$$0 \leqslant \zeta \leqslant \zeta_m, \quad P(\zeta) = P_m - G_b\left[\operatorname{Ei}\left(-\frac{\zeta}{4\varkappa^*}\right) - \operatorname{Ei}\left(-\frac{\zeta_m}{4\varkappa^*}\right)\right] \tag{3.4}$$

$$P_m = 1 + \frac{\delta_{1m} - 1}{B_p}, \quad \delta_{1m} = \delta_2\left[1 - \frac{4G_b}{\zeta_m}\exp\left(-\frac{\zeta_m}{4\varkappa^*}\right)\right]^{-1} \tag{3.5}$$

The quantity ζ_m will be determined below.

As we have already said, the fact that $\beta_p = \Delta p \ll 1$ implies that in order to find the temperature fields $\theta(\zeta)$, we can assume that $\rho_1 = \rho_{10}$ in (1.3), i.e. we can neglect the change in density of the liquid (molten) phase caused by a change in the pressure. Then from (1.3), (1.14), (2.1) and (1.16)-(1.18), taking (1.2) and (3.4) into account, we obtain the following ordinary differential equations and boundary conditions:

$$\frac{d}{d\zeta}\left(\zeta\frac{d\theta}{d\zeta}\right) + \left[\frac{\zeta}{4a_i^*} + V_k(\zeta)\right]\frac{d\theta}{d\zeta} = -Q_i^*(\zeta) \quad (k = l, s) \tag{3.6}$$

$$\zeta = 0, \ \theta = \theta_b \text{ or } \zeta d\theta/d\zeta = -q_b^* \tag{3.7}$$

$$(q_b^* = q_b/(2\chi(\nu)\lambda_l))$$

$$\theta|_{\zeta=\zeta_m-0} = \theta|_{\zeta=\zeta_m+0} = 1 \tag{3.8}$$

$$-G_l\,d\theta/d\zeta|_{\zeta=\zeta_m-0} + G_s\,d\theta/d\zeta|_{\zeta=\zeta_m+0} = m/4 \tag{3.9}$$

$$\zeta \to +\infty, \ \theta \to \theta_0 \tag{3.10}$$

$$0 \leqslant \zeta \leqslant \zeta_m, \quad a_k^* = a_l^* = 1, \quad V_i(\zeta) = V_l(\zeta) = \tag{3.11}$$

$$-MG_b\exp\left(-\frac{\zeta}{4\varkappa^*}\right), \quad M = mB_p = \Phi_1\varkappa^*$$

$$Q_i^*(\zeta) = Q_l^*(\zeta) = \frac{K_l\zeta^{-1/2}}{4h_l^*}\exp\left(-\frac{\sqrt{\zeta}}{h_l^*}\right)\times$$

$$\left\{1 + H\exp\left[-\frac{2}{h_l^*}(\sqrt{\zeta_m} - \sqrt{\zeta})\right]\right\}$$

$$\zeta > \zeta_m, \quad a_k^* = a_s^*, \quad V_k(\zeta) = V_s(\zeta) = 0 \tag{3.12}$$

$$Q_i^*(\zeta) = Q_s^*(\zeta) = (1 - H)\frac{K_s\zeta^{-1/2}}{4h_s^*}\exp\left(-\frac{\sqrt{\zeta_m}}{h_l^*} + \frac{\sqrt{\zeta_m} - \sqrt{\zeta}}{h_s^*}\right)$$

The solutions of these equations have the form

$$0 \leqslant \zeta \leqslant \zeta_m, \quad \theta(\zeta) = I_l(\zeta) + C_l J_l(\zeta) + D_l \tag{3.13}$$

$$\left(I_l(\zeta) = -\int_{\zeta}^{\zeta_m} F_l(u)X_l(u)\,du, \quad J_l(\zeta) = \int_{\zeta}^{\zeta_m} X_l(u)\,du\right.$$

$$\left.F_l(u) = \int_u^{\zeta_m}\frac{Q_l^*(\xi)}{X_l(\xi)}d\xi, \quad X_l(u) = \frac{1}{u}\exp\left[MG_b\operatorname{Ei}\left(-\frac{u}{4\varkappa^*}\right) - \frac{u}{4}\right]\right)$$

$$\zeta > \zeta_m, \quad \theta(\zeta) = I_s(\zeta) + C_s J_s(\zeta) + D_s \tag{3.14}$$

$$\left(I_s(\zeta) = -\int_{\zeta_m}^{\zeta} F_s(u) X_s(u)\, du, \quad J_s(\zeta) = \int_{\zeta_m}^{\zeta} X_s(u)\, du\right.$$
$$\left. F_s(u) = \int_{\zeta_m}^{u} \frac{Q_s^*(\xi)}{X_s(\xi)} d\xi, \quad X_s(u) = \frac{1}{u} \exp\left(-\frac{u}{4a_s^*}\right)\right)$$

where C_l, D_l, C_s and D_s are the constants of integration determined by the boundary conditions at $\zeta = 0$, $\zeta = \zeta_m$ and $\zeta \to +\infty$ given in (3.7), (3.8) and (3.10). This shows that for the problem in question the dimensionless similarity criteria are represented by the parameters (3.2) characterizing, respectively, the ratio of the heat emission to the heat conduction (K_l and K_s), the influence of the thermophysical property of the phases (M and a_s^*), the ratio of the piezoconductivity to the thermal conductivity ($\varkappa^*$), the influence of the mass transport (G_b), the effect of the electrophysical properties of the medium (h_l^*, h_s^* and H), the influence of the melting process (G_l and G_s; see (4.9) below), and also the parameters θ_b or q_b^*, θ_0 and m appearing in the boundary conditions. A total of 14 parameters in all determines the set of selfsimilar solutions in question.

4. It can be shown that the following asymptotic expressions hold as $\zeta \to +0$:

$$X_l(\zeta) \sim \zeta^{-1+\alpha}, \quad F_l(\zeta) \sim \int_{\zeta} u^{-1/2-\alpha}\, du$$
$$x\frac{\partial T}{\partial x} \sim \zeta \frac{d\theta}{d\zeta} \sim \zeta^{\alpha}, \quad \alpha = MG_b \tag{4.1}$$

When $\zeta \to +\infty$, the integrals in (3.14) converge. The existence of solutions and the solution itself of the problem in question, depend on the sign of G_b and the type of boundary conditions.

a) For $G_b > 0$ when the molten material flows from the well $(v_1 > 0)$ and a boundary condition of the first kind is specified at the well

$$\zeta = 0, \ \theta = \theta_b = \text{const} > 1 \tag{4.2}$$

the selfsimilar solution of the problem in question exists and has the form

$$0 \leqslant \zeta \leqslant \zeta_m, \quad \theta(\zeta) = 1 + I_l(\zeta) + [\theta_b - 1 - I_l(0)] \frac{J_l(\zeta)}{J_l(0)} \tag{4.3}$$
$$\zeta > \zeta_m, \ \theta(\zeta) = 1 + I_s(\zeta) + [\theta_0 - 1 - I_s(+\infty)] - J_s(\zeta)/J_s(+\infty)$$

The pressure field in the molten (first) phase is determined using relation (3.4). From (4.3) it follows that

$$q_b^* = -\zeta\,(d\theta/d\zeta)_{\zeta=0} = 0$$

When a boundary condition of the second kind (with a finite, non-zero heat flux) is specified, then for $\zeta = 0$ the problem has no selfsimilar solution.

b) For $G_b = 0$, the molten phase of the second component is not in motion $(v_1 = 0)$ and a finite boundary condition of the first kind (finite temperature) is specified at the well, the problem has no selfsimilar solutions. When a boundary condition of the second kind is specified at the well

$$\zeta = 0, \ \zeta\,(d\theta/d\zeta) = -q_b^* \tag{4.4}$$

we have a generalization of the selfsimilar solution of the classical Stefan problem (without convective heat transport), taking into account the volume heat source

$$0 \leqslant \zeta \leqslant \zeta_m, \ \theta(\zeta) = 1 + I_l(\zeta) + [q_b^* + F_l(0)]\, J_l(\zeta) \tag{4.5}$$
$$\zeta > \zeta_m, \ \theta(\zeta) = 1 + I_s(\zeta) + [\theta_0 - 1 - I_s(+\infty)]\, J_s(\zeta)/J_s(+\infty)$$

In this case the temperature field $\theta(\zeta)$ as $\zeta \to +0$, tends to infinity as $\ln(1/\zeta)$.

c) For $G_b < 0$, the molten material moves towards the well $(v_1 < 0)$ and the finite temperature or finite heat flux is specified for $\zeta = 0$, the problem has no selfsimilar solutions. However in this case we can use (3.13) and (3.14) to seek a solution for the case when a generalized boundary condition

$$\zeta \to +0, \ \zeta\,(d\theta/d\zeta) \sim -q_b^* \zeta^{\alpha}, \ \alpha = MG_b \tag{4.6}$$

is specified, where q_b^* is a given constant determined by the asymptotic behaviour of the incoming heat due to thermal conduction at the well. Then the temperature fields will have the form

$$0 \leqslant \zeta \leqslant \zeta_m, \quad \theta(\zeta) = 1 + I_l(\zeta) + [q_b^* \exp(-MG_bB) + F_l(0)] J_l(\zeta) \tag{4.7}$$

$$(B = \lim_{u \to +0} [\mathrm{Ei}(-u) - \ln(u)])$$

$$\zeta > \zeta_m, \quad \theta(\zeta) = 1 + I_s(\zeta) + [\theta_0 - 1 - I_s(+\infty)] J_s(\zeta)/J_s(+\infty)$$

The solution (4.7) is a generalization of (4.5) for the case $G_b < 0$. Formally, (4.5) follows from the solution (4.7) as $G_b \to 0$. In this case the temperature and the heat flux both tend to infinity as $\zeta \to +0$.

In order to determine the parameter ζ_m which provides the law of motion of the melting front in the form

$$x_m(t) = \sqrt{\zeta_m a_l t}$$

we obtain, from the equation of energy balance at the interphase boundary $x_m(t)$ (3.9), taking into account (3.13) and (3.14), the equation

$$G_lC_l(\zeta_m) X_l(\zeta_m) + G_sC_s(\zeta_m) X_s(\zeta_m) = m/4$$

For porous media we have, as a rule, $\varkappa \gg a_l$. Therefore, remembering that $\varkappa^* \to +\infty$, we can simplify the relation $X_l(u)$ in the course of determining the temperature distribution (see (3.13)) thus:

$$X_l(u) = u^{-1+\alpha} \exp(-u/4)$$

This approximate distribution of the temperature of the medium corresponds to the solution of the equation of heat conduction (3.6) with the following velocity field of the molten material: $V_l(\zeta) = -MG_b/\zeta$, which corresponds to the solution of the equation of piezo-conduction (1.8) in the quasistationary approximation $(\partial p_1/\partial t = 0)$ when the pressure field can be represented in the form $P(\zeta) = P_m - G_b \ln(\zeta/\zeta_m)$.

Let us analyse the solutions (4.3), (4.5) and (4.7). We obtain from (4.3), after differentiating, i.e. when $G_b > 0$ and $v_1 > 0$,

$$0 \leqslant \zeta \leqslant \zeta_m, \quad \frac{d\theta}{d\zeta} = X_l(\zeta)\left[F_l(\zeta) - \frac{\theta_b - 1 - I_l(0)}{J_l(0)}\right]$$

This implies that when no EW energy is absorbed by the medium $(Q = 0)$, the function $\theta(\zeta)$ in the region occupied by the molten phase of the second component $(0 \leqslant \zeta \leqslant \zeta_m)$ is concave and decreases uniformly from θ_b to 1. When a volume heat source is present $(Q > 0)$, the function $\theta(\zeta)$ may become convex and non-monotonic. Moreover, $\theta(\zeta)$ cannot have more than one maximum.

Fig.2 shows the characteristic temperature profiles of the medium $\theta(\zeta)$ b in the region occupied by the molten material $(0 \leqslant \zeta \leqslant \zeta_m)$ for $G_b > 0$ and various intensities of the volume heat source Q. Curve *1* corresponds to the case $Q = 0$. When Q increases, e.g. as a result of an increase in $N^{(e)}$, a maximum appears in the distribution $\theta(\zeta)$ and the convexity of $\theta(\zeta)$ becomes more pronounced (curve *3*). Curves *2* and *3* in Fig.2 correspond to the case $Q > 0$. In all cases the melting front corresponds to the point $\zeta_m = x_m^2/a_l t$.

It can be shown that the solutions (4.5), (4.7) are physically meaningful only when $q_b \geqslant 0$, since otherwise from (4.5) and (4.7) it follows that when $\zeta \to +0$, the temperature $\theta(\zeta)$ becomes negative and tends to infinity as $\ln\zeta$ and $-\zeta^\alpha$ $(\alpha = MG_b)$ respectively.

However, in this, and other such cases, the solutions of the problem can be studied using a two-front formulation /9/.

Fig.3 shows characteristic profiles of the temperature fields and of the medium $\theta(\zeta)$ in the region occupied by the molten material $(0 \leqslant \zeta \leqslant \zeta_m)$ for the cases $G_b = 0$ and $G_b < 0$ (the first relations in the solutions (4.5) and (4.7)). When $q_b = 0$ $(Q > 0)$, the temperature of the medium at the centre $\theta(0)$ is finite (curve *1*) in both cases. When $q_b > 0$ $(Q \geqslant 0)$ the temperature tends to $+\infty$ (curve *2* in Fig.3).

5. We find that within the model of discontinuity adopted here, separating the solid (non-molten) and liquid (molten) phase of the second component, the thermohydrodynamic processes within the region occupied by the liquid phase $(0 \leqslant \zeta \leqslant \zeta_m)$ affect the heating process within the solid phase $(\zeta > \zeta_m)$ only through the dynamics of the melting front $x_m(t)$, namely through the parameter ζ_m. We see from the solutions (4.3), (4.5) and (4.7) that the temperature distributions of the medium in the region occupied by the solid second component $(\zeta > \zeta_m)$ have the same form for all three cases $(G_b > 0,\ G_b = 0$ and $G_b < 0)$. The solutions also imply that in the region $\zeta > \zeta_m$

$$\frac{d\theta}{d\zeta} = X_s(\zeta)\left[-F_s(\zeta) + \frac{\theta_0 - 1 - I_s(+\infty)}{J_s(+\infty)}\right] \qquad (5.1)$$

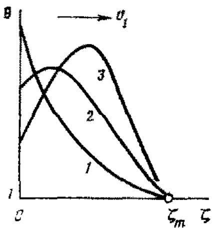

Fig.2

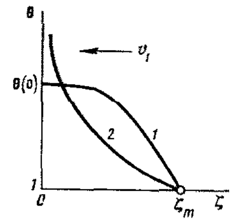

Fig.3

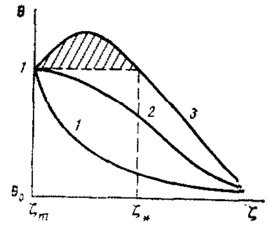

Fig.4

From this we can obtain the condition for non-existence of a zone of superheated solid (non-molten) phase of the second component (the condition that the temperature field of the medium $\theta(\zeta)$ does not increase with respect to ζ when $\zeta \geqslant \zeta_m$)

$$I_s(+\infty) + 1 - \theta_0 \geqslant 0 \quad (I_s(+\infty) \leqslant 0) \qquad (5.2)$$

When no EW energy is absorbed by the medium $(Q = 0)$, we have $I_s(+\infty) = 0$. Therefore the condition (5.2) always holds by virtue of $\theta_0 \leqslant 1$. In the presence of a volume heat source $(Q > 0)$ we have $I_s(+\infty) < 0$. The quantity $I_s(+\infty)$ decreases as Q increases, and this can lead to violation of condition (5.2) and hence to the loss of the monotonic form of the temperature field of the medium in the region $\zeta \geqslant \zeta_m$, i.e. to the appearance of a zone of superheated solid second component $(\zeta_m < \zeta < \zeta_*,\ \theta(\zeta) > 1$ where $\theta(\zeta_*) = 1)$.

Fig.4 shows the characteristic profiles of the temperature fields of the medium $\theta(\zeta)$ in the region $\zeta > \zeta_m$ for the different intensities of the volume heat source Q. Curve *1* corresponds to the case $Q = 0$, curve *2* to the case when there is no zone of superheated solid second component (second phase): $\theta(\zeta) \leqslant 1$, and curve *3* corresponds to the case when there is a zone of superheated second phase: $\zeta_m < \zeta < \zeta_*,\ \theta(\zeta) > 1 \quad (I_s(+\infty) + 1 - \theta_0 < 0)$ (in the last two cases $Q > 0$). In (x, t) coordinates the size of the zone of superheated second phase increases with time according to the law $(\sqrt{\zeta_*} - \sqrt{\zeta_m})\sqrt{a_l t}$. When $\theta_0 = 1$, we see from (3.13) and (5.2) that for any $Q > 0$ the zone of superheated second phase will be of infinite size since in this case we have for any $\zeta > \zeta_m,\ \theta(\zeta) > 1$.

It should be noted that the presence of a zone of superheated second phase (non-molten second component) implies that the model of discontinuity (Stefan scheme) separating the solid (non-molten) and liquid (molten) phase of the second component cannot be used to describe the phenomenon in question. In this case, in order to carry out a mathematical modelling of the process we must replace the melting surface by a two-phase zone of melting, of non-zero thickness. The liquid and solid phase of the second component will exist simultaneously within this zone $(\alpha_1\alpha_2 \neq 0)$ and the temperature will be equal to the melting temperature $(\theta = 1)$.

6. Selfsimilar solutions of a more specific system of differential equations of heat conduction and filtration of a molten material without volume heating $(Q = 0)$ were studied in /8/ for the plane, and in /10/ for the axisymmetric case. The selfsimilar solution of this problem was studied, taking into account the convective heat transport caused, as in Sect. 2-5, by the difference in the densities of the solid and molten phase of the second component. The selfsimilar solution of the problem without filtration $(v_1 = 0)$ and without convective heat transport in the equation of heat conduction, was obtained in /11, 12/. However, an erroneous equation of energy balance at the interphase boundary used in /8/ (see the discussion of Eq. (1.19)) has led to an incorrect value of ζ_m, and hence to an incorrect determination of the law of motion of the melting front $x_m(t) = \sqrt{\zeta_m a_l t}$. The solution of the equation of heat conduction in the molten phase zone of the second component (see (3.6), (3.11)) obtained in /10/ for the case $G_b < 0$ and $Q_l^* = 0$, is also incorrect. Moreover, the problem in the mode shown, when the molten material flows in the well and the final boundary conditions of any type (the final temperature or final heat flux) are given, has no solution when $\zeta = 0$. The analysis of the existence of the solution of the problem and the solution itself, when $Q = 0$, for the cases $G_b < 0$, $G_b = 0$ and $G_b > 0$ when follows from Sect.2-5 of this paper when $Q_l^*(\zeta) = Q_s^*(\zeta) = 0 \ (I_l(\zeta) = I_s(\zeta) = 0)$.

7. We can single out two limiting cases. In the first case the second component is in the liquid state everywhere $(\alpha_1 = m > 0,\ \alpha_2(x, t) = 0)$. Then we must put $\zeta_m = +\infty$ in the solution in question, and we have

$$F_l(u) = \int_u^1 X_l^{-1}(\xi) Q_l^*(\xi)\, d\xi$$

When a finite pressure is given at infinity, the problem has no solution in the quasi-stationary approximation $(\partial p_1/\partial t = 0)$ for $G_b \neq 0\ (v_1 \neq 0)$. When $G_b = 0\ (v_1 = 0)$, we have a trivial stationary pressure field $P(\zeta) = P_m = P_0 = 1\ (P_0 = P(\zeta \to \infty))$.

If we take into account the dependence of the viscosity of the liquid phase of the second component on temperature, the temperature and filtration equations will depend on each other and the equation of piezoconduction (1.8) will become non-linear.

In the second limiting case there is no liquid phase of the second component and no melting $\alpha_1(x, t) = 0,\ \alpha_2 = m,\ v_1 = 0,\ G_b = 0,\ \zeta_m = 0$, and the temperature distribution is given by (3.14). Here we have

$$F_s(u) = \int_1^u X_s^{-1}(\xi) Q_s^*(\xi)\, d\xi$$

The selfsimilar solutions of the resulting system of differential equations can be used to analyse the characteristic features of the phenomenon in question, to assess the heating of the medium, the velocity of motion of the melting front, etc. They can also be used to test the validity of various approximations and numerical solutions of the complete system of equations.

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